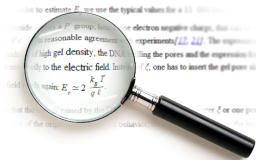


Maths Information Retrieval for Digital Libraries

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<https://mir.fi.muni.cz/>

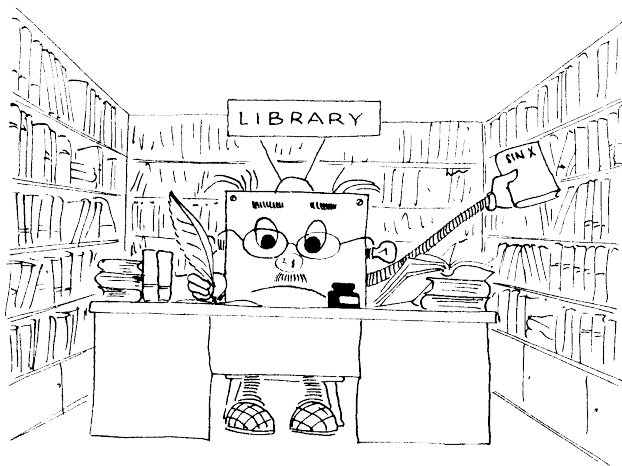


Illustrations by Jiří Franek.

Outline

- 1 Motivation
- 2 Math Indexer and Searcher
- 3 MathML Canonicalizer
- 4 My Goals
- 5 Evaluation
- 6 Future Works
- 7 Summary

Motivation



Motivation

- *Czech Digital Mathematics Library* (DML-CZ), <http://www.dml.cz/>
- *Centre de diffusion de revues académiques mathématiques* (CEDRAM), <http://www.numdam.org/>
- *Numérisation de documents anciens mathématiques* (NUMDAM), <http://www.cedram.org/>
- *Göttingen Göttinger Digitalisierungszentrum* (GDZ), <http://gdz.sub.uni-goettingen.de/>
- *Electronic Research Archive for Mathematics* (ERAM), <http://www.emis.de/projects/JFM/>
- *The Electronic Library of Mathematics* (ELibM), <http://siba-sinmemis.unile.it/ELibM.html>
- *Journal STORage* (JSTOR), <http://www.jstor.org/>
- *Project Euclid*, <http://projecteuclid.org/>
- *Russian Digital Mathematics Library* (RusDML), <http://www.rusdml.de/>
- *Polish Digital Mathematical Library* (DML-PL), <http://pldml.icm.edu.pl/>
- *Biblioteca Digital Española de Matemáticas* (DML-E), <http://dmle.cindoc.csic.es/>
- *Japanese Digital Mathematics Library* (DML-JP), <http://sparc1.math.sci.hokudai.ac.jp/dmljp/>
- *Riviste Elettroniche Italiane di Matematica* (REIM), <http://siba2.unile.it/sinm/reim/>
- *Biblioteca Digitale Italiana di Matematica* (bdim), <http://www.bdim.eu/>

Motivation (cont.)

Q: 'What functionality and incentives would made a working mathematician to login and use a modern DML as EuDML?'

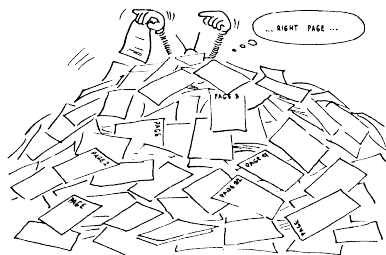
A: '**Math formulae search.**'

Prof. James Davenport, CEIC member, MKM 2011 PC chair, on panel at DML 2011 workshop in Bertinoro as a reply.



Motivation (cont.)

- DML without maths-aware search support is an oxymoron.



- Simple search based on text keywords is not appropriate or sufficient for mathematical contents.

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Math Indexer and Searcher (MIaS)



Match of the following rules

[remove](#)

[Add clause](#)

Contains the following formula:

k/H_0^{2S}

Math formulae can be entered either in TeX or MathML notation (format will be autodetected). LaTeX math has to be enclosed within $...$. AMS packages are supported.

Rendered: k/H_0^2

[Search](#)

Search using:

Search in:

Verbose output:

Total hits: 16, showing 1-16 . Core searching time: 1379 ms Total searching time: 2664 ms

[Giant Vortex Lattice Deformations in Rapidly Rotating Bose-Einstein Condensates](#)

... suggesting the **vortex density** ℓ/R_0^2 to be the dominant factor determining the variation of giant **vortex** core oscillation frequencies. ... (larger ℓ/R_0^2) the core oscillates rapidly, but with increasing core size, the oscillation frequency slows, and approaches the value of the breathing mode, as the size of the giant **vortex** approaches that of the condensate itself. Giant Vortex Lattice Deformations in Rapidly Rotating Bose-Einstein Condensates ... We have performed numerical simulations of giant **vortex** structures in rapidly rotating Bose-Einstein condensates within the Gross-Pitaevskii formalism.

score = 1.2043247

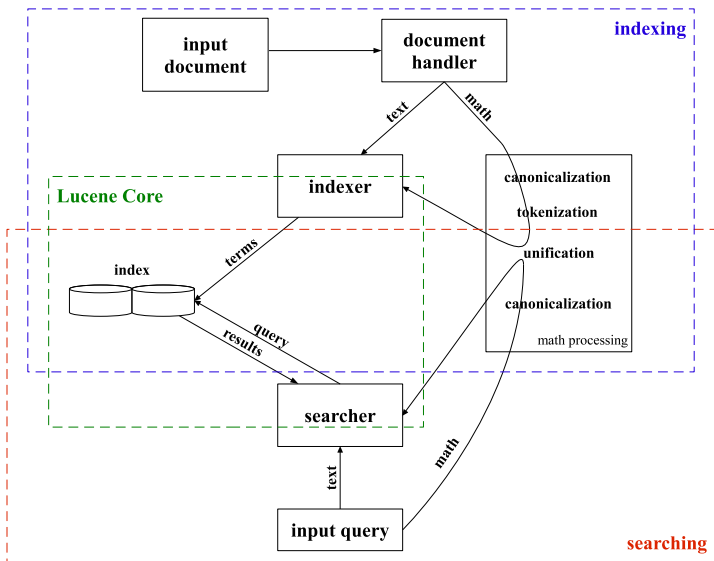
<http://arxiv.org/abs/cond-mat/0307130> - cached XHTML

[Split-merge cycle, fragmented collapse, and vortex disintegration in rotating Bose-Einstein condensates with attractive interactions](#)

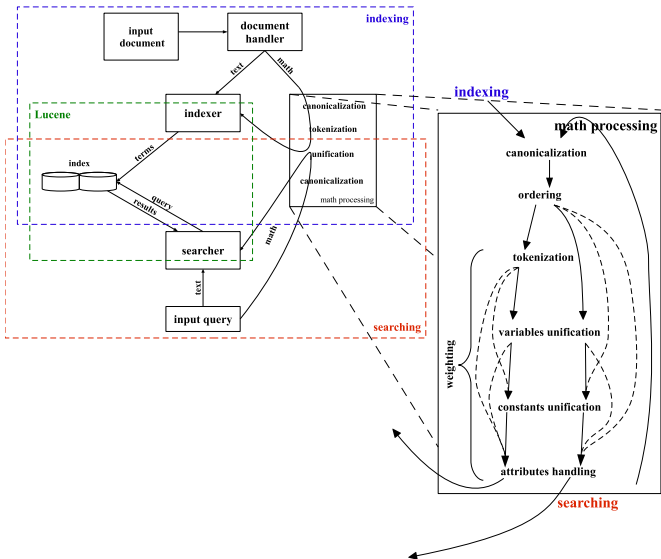
... and $(N|a_0^+|)^{1/2}$, respectively, where ... Split-merge cycle, fragmented collapse, and **vortex** disintegration in rotating Bose-Einstein condensates with attractive interactions ... The dynamical instabilities and ensuing dynamics of singly- and doubly-quantized **vortex** states of Bose-Einstein condensates with attractive interactions are investigated using full 3D numerical simulations of the Gross-Pitaevskii equation. ... -fold **density** modulation grows exponentially in time while rotating at frequency ...

----- 6 0307130

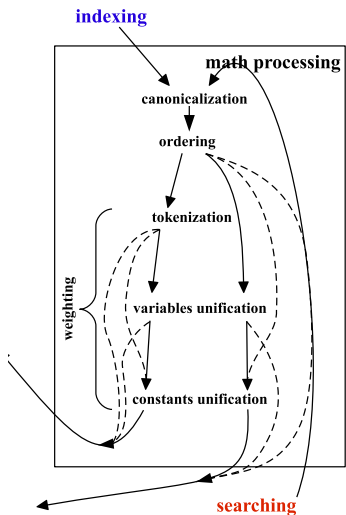
MIAS Design



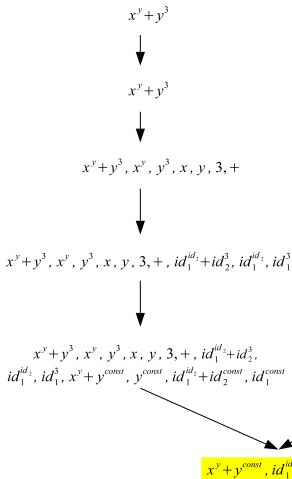
MlaS Design



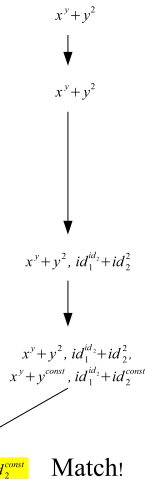
MIAS Indexing and Searching Example



indexing



searching



$x^y + y^{const}, id_1^{id_2} + id_2^{const}$

Match!

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Sources of MathML in Digital Libraries

- ‘Hand made’
- Tralics
- L^AT_EXML
- InfyReader
- MaxTract
- MATLAB
- Wolfram
Alpha
- ...

‘Hand made’ $x^2 + y^2$ MathML

```
<math xmlns='http://www.w3.org/1998/Math/MathML'>  
  <msup>  
    <mi>x</mi><mn>2</mn>  
  </msup>  
  <mo>+</mo>  
  <msup>  
    <mi>y</mi><mn>2</mn>  
  </msup>  
</math>
```

Sources of MathML in Digital Libraries

- ‘Hand made’
- Tralics
- \LaTeX XML
- InfyReader
- MaxTract
- MATLAB
- Wolfram
Alpha
- ...

Matlab $x^2 + y^2$ MathML

```
generate::MathML(x^2 + y^2,
                  Content = FALSE, Annotation = FALSE)
<math xmlns='http://www.w3.org/1998/Math/MathML' >
  <mrow xref='No7' >
    <msup xref='No3' >
      <mi xref='No1' >x</mi>
      <mn xref='No2' >2</mn>
    </msup>
    <mo>+</mo>
    <msup xref='No6' >
      <mi xref='No4' >y</mi>
      <mn xref='No5' >2</mn>
    </msup>
  </mrow>
</math>
```

Sources of MathML in Digital Libraries

- ‘Hand made’
- Tralics
- \LaTeX XML
- InftyReader
- MaxTract
- MATLAB
- Wolfram
Alpha
- ...

\LaTeX XML $x^2 + y^2$ MathML

```
<math xmlns="http://www.w3.org/1998/Math/MathML"
  alttext="x^{2}+y^{2}" display="inline">
  <semantics>
    <mrow>
      <msup><mi>x</mi><mn>2</mn></msup>
      <mo>+</mo>
      <msup><mi>y</mi><mn>2</mn></msup>
    </mrow>
    <annotation encoding="application/x-tex">
      x^{2}+y^{2}
    </annotation>
  </semantics>
</math>
```

Sources of MathML in Digital Libraries

- ‘Hand made’
- Tralics
- L^AT_EX XML
- InftyReader
- MaxTract
- MATLAB
- Wolfram Alpha
- ...

InftyReader $x^2 + y^2$ MathML

```
<math xmlns="http://www.w3.org/1998/Math/MathML">
  <msup>
    <mi mathvariant="italic">x</mi>
    <mrow>
      <mn mathvariant="normal">2</mn>
    </mrow>
  </msup>
  <mo mathvariant="normal">+</mo>
  <msup>
    <mi mathvariant="italic">y</mi>
    <mrow>
      <mn mathvariant="normal">2</mn>
    </mrow>
  </msup>
</math>
```


MathML Canonicalizer

- Our own MathML canonicalization tool.
- The main design imperatives:
 - Modularity,
 - simplicity,
 - extensibility, and
 - flexibility.
- The speed of the canonicalization application is also a critical parameter.
 - In our MREC corpora there is 168,000,000 formulae to canonicalize.

MathML Canonicalizer Use Cases

<mphantom> Omission

```
<mfrac>
  <mrow>
    <mi> x </mi>
    <mo> + </mo>
    <mi> y </mi>
    <mo> + </mo>
    <mi> z </mi>
  </mrow>
  <mrow>
    <mi> x </mi>
    <mphantom>
      <mo> + </mo>
      <mi> y </mi>
    </mphantom>
    <mo> + </mo>
    <mi> z </mi>
  </mrow>
</mfrac>
```

```
<mfrac>
  <mrow>
    <mi> x </mi>
    <mo> + </mo>
    <mi> y </mi>
    <mo> + </mo>
    <mi> z </mi>
  </mrow>
  <mrow>
    <mi> x </mi>
    <mo> + </mo>
    <mi> z </mi>
  </mrow>
</mfrac>
```

MathML Canonicalizer Use Cases

Unnecessary Attributes

```
<mfrac linethickness="2"  
      bevelled="true">  
  <mi> a </mi>  
  <mi> b </mi>  
</mfrac>
```

```
<mfrac>  
  <mi> a </mi>  
  <mi> b </mi>  
</mfrac>
```

MathML Canonicalizer Use Cases

<mrow> Minimizing

```
<msqrt>
```

```
<mrow>
```

```
<mo> - </mo>
```

```
<mn> 1 </mn>
```

```
</mrow>
```

```
</msqrt>
```

```
<msqrt>
```

```
<mo> - </mo>
```

```
<mn> 1 </mn>
```

```
</msqrt>
```

MathML Canonicalizer Use Cases

Unifying Fences

```
<mfenced open="["">
```

```
  <mi> x </mi>
```

```
  <mi> y </mi>
```

```
</mfenced>
```

```
<mrow>
```

```
  <mo> [ </mo>
```

```
  <mrow>
```

```
    <mi> x </mi>
```

```
    <mo> , </mo>
```

```
    <mi> y </mi>
```

```
  <mrow>
```

```
    <mo> ) </mo>
```

```
</mrow>
```

MathML Canonicalizer Use Cases

Sub-/Superscripts Handling

```
<msubsup>  
  <mi> x </mi>  
  
  <mn> 1 </mn>  
  
  <mn> 2 </mn>  
</msubsup>
```

```
<msup>  
  <msub>  
    <mi> x </mi>  
    <mn> 1 </mn>  
  </msub>  
  <mn> 2 </mn>  
</msup>
```

MathML Canonicalizer Use Cases

Applying Functions

```
<mi> f </mi>  
<mo> &#x2061; </mo>  
<mrow>  
  <mo> ( </mo>  
  <mi> x </mi>  
  <mo> ) </mo>  
</mrow>
```

```
<mi> f </mi>  
<mrow>  
  <mo> ( </mo>  
  <mi> x </mi>  
  <mo> ) </mo>  
</mrow>
```

MathML Canonicalizer Use Cases

Applying Functions

```
<mi> sin </mi>
```

```
<mo> &#x2061; </mo>
```

```
<mi> x </mi>
```

```
<mi>sin</mi>
```

```
<mrow>
```

```
  <mo>( </mo>
```

```
  <mi>x</mi>
```

```
  <mo>)</mo>
```

```
</mrow>
```


MathML Canonicalizer Web Evaluation Application

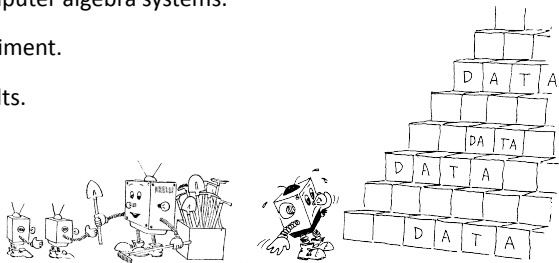
- JUnit testing seems not to be enough.
- Collaboration-enabled evaluation system needed:
 - Visualization of test data collection.
 - History of canonicalization results on the same data with different version of the Canonicalizer.
 - Annotations on the results.
 - Statistics.
 - Coverage of all the mark-up of the MathML standard.

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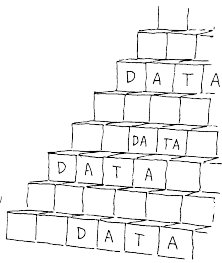
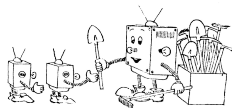
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Ranking of the results.



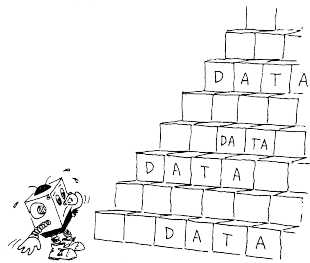
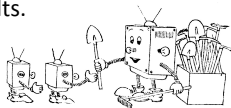
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Canonicalization of both Presentation and Content MathML.
 - Classification of identifiers.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Ranking of the results.



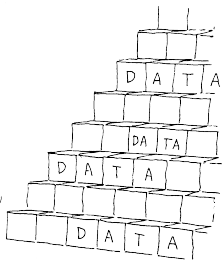
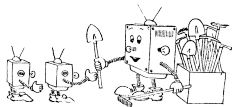
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - To mark particular identifiers as variable name, function name, and so on.
 - Start from the metadata available for the documents.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Ranking of the results.



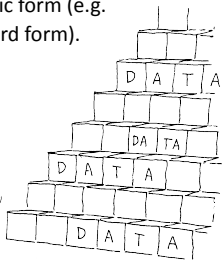
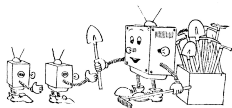
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - Context driven search.
 - Exploitation of the metadata available for the documents.
 - Exploitation of the users' inputs.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Ranking of the results.



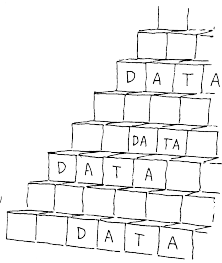
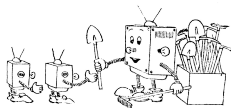
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Manipulation of mathematical expressions in symbolic form (e.g. simplification to a smaller expression or some standard form).
 - Image search experiment.
 - Ranking of the results.



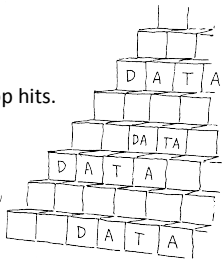
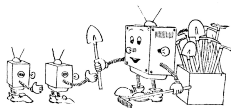
Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Experiments with visual similarity.
 - Ranking of the results.



Objectives

- Improvements of the relevance of the results of our math-aware search engine.
 - MathML Normalization.
 - Classification of identifiers.
 - Context driven search.
 - Involvement of computer algebra systems.
 - Image search experiment.
 - Ranking of the results.
 - Score bonus for documents to be known similar to top hits.
 - Internal subqueries for a single user's query.



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NTCIR-10 Math Task

- The first (pilot) year of the math task event last year (i.e. 2013).
- Formula search and Full-text search.
 - 4 runs submitted – differ in query language.
 - PMath – Run #1.
 - CMath – Run #2.
 - PCMath – Run #3.
 - T_EX – Run #4.
- Open Information Retrieval.
 - 1 run submitted – T_EX + text mixed queries.

NTCIR-10 Math Task Results

Table 1: Result metrics for submitted runs in Formula Search with Relevance Level ≥ 3 (Relevant)

Metric	Run 1	Run 2	Run 4
P-10 avg	0.105	0.191	0.219
P-5 avg	0.133	0.229	0.276
MAP avg	0.060	0.112	0.127
Precision	0.109 (64/589)	0.185 (92/496)	0.123 (96/778)

Table 2: Result metrics for submitted runs in Formula Search with Relevance Level ≥ 1 (Partially Relevant)

Metric	Run 1	Run 2	Run 4
P-10 avg	0.143	0.214	0.267
P-5 avg	0.181	0.267	0.343
MAP avg	0.066	0.081	0.100
Precision	0.148 (87/589)	0.232 (115/496)	0.161 (125/778)

NTCIR-11 Math Task

- A greater number of participants.
 - Increase from 6 to 8.
- Only one type of queries.
 - 50 queries, each
 - 1–4 formulae,
 - 1–4 keyphrases.
- Our results submitted for judgement in June 2014.

NTCIR-11 Math Task: Our Investigation of Our Results

Improper query conversion

Index: $\operatorname{Im}P^{+}_{\Gamma}=C_{\mu}^{+}(\Gamma)$

Query: $\operatorname{Im}P^{+}_{\gamma}=C^{+}_{\mu}(\gamma)$

Index

```

...
<mrow>
  <mo>Im</mo>
  <mo></mo>
  <msup>
...

```

Query

```

...
<mrow>
  <mi>I</mi>
  <mi>m</mi>
  <msup>
...

```

NTCIR-11 Math Task: Our Investigation of Our Results

Substructure difference tolerance should be improved

```
...  
<mrow>  
  QUERY-FORMULA-SUBPART-1  
  [[ INDEX  
    <mrow>  
      <mo>f</mo>  
    <mrow>  
  || QUERY  
    <mi>o</mi>  
  ]]  
  QUERY-FORMULA-SUBPART-2  
  INDEX-FORMULA-SUPPLEMENT  
    </mrow>  
  </mrow>  
</mrow>  
</mrow>  
...
```


NTCIR-11 Math Task: Our Investigation of Our Results

$\backslash\text{qvar}\{\}$ handling

Original task query:

$$\backslash\text{qvar}\{S\} = -\backslash\text{qvar}\{T\}_{\backslash\text{qvar}\{p\}} \backslash\text{int} \backslash\text{qvar}\{d\}^{\backslash\text{qvar}\{p\}+1} \backslash\text{qvar}\{x\} \backslash\text{sqrt}\{\backslash\text{qvar}\{g\}\}$$

Index: $S = -T_{\{p\}} \backslash\text{int} d^{\{p+1\}} x \backslash\text{sqrt}\{-g\}$

Query: $S = -T_{\{p\}} \backslash\text{int} d^{\{p+1\}} x \backslash\text{sqrt}\{g\}$

$\backslash\text{sqrt}\{g\}$ does not match $\backslash\text{sqrt}\{-g\}$.

NTCIR-11 Math Task: Our Investigation of Our Results

Unification would be helpful

Query:

$\backslash\text{qvar}\{x\}\backslash\text{frac}\{\backslash\text{qvar}\{y\}\}\{\backslash\text{qvar}\{z\}\}-\backslash\text{qvar}\{u\}\backslash\text{frac}\{\backslash\text{qvar}\{v\}\}\{\backslash\text{qvar}\{w\}\}$

$$x\frac{y}{z} - u\frac{v}{w}$$

Matches:

$$\begin{aligned} \{q_s, q_r\} &= \int dx \int dy \{A(x, \mu)^p, A(y, \nu)^q\} \Big|_{\mu^{s+1}\nu^{r+1}} \\ &= pq\mu\nu \int dx A(\mu)^p (A(\nu)^q)' \left[\left\{ \frac{s}{p}\nu - \frac{r}{q}\mu \right\} \frac{1}{\mu-\nu} + \frac{1}{h} \frac{rs}{pq} \right] \Big|_{\mu^{s+1}\nu^{r+1}} \end{aligned}$$

Does not match:

$$\zeta \sim c_1 \frac{\delta\rho_\sigma}{\rho_\sigma} - c_2 \frac{\delta H_{\text{osc}}}{H_{\text{osc}}}$$

NTCIR-11 Math Task: Our Investigation of Our Results

- Combination of both formulae and text keywords in one query is important.
- Multiple subqueries derived from the original query with result lists merging turned out to be very useful.
 - One-by-one removal of the keywords and formulae.
 - “Strip-merging” of the results of the subqueries.
 - Further investigation of the best strategies needed.
 - Subqueries with subformulae?
 - Different strategy for merging of results?

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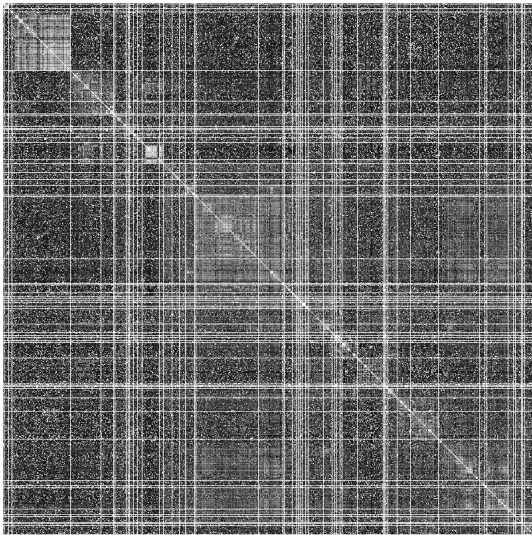
Future Works

- Long-term goals:
 - All the objectives mentioned earlier. 😊
 - Whole arXiv indexing for MlaS search.
 - Evaluation.
- Short-term goals:
 - Improved strategies for MlaS internal subqueries derived from a single user's query.
 - Integration of MlaS with DML-CZ DSpace.
 - Exploitation of Gensim-Math-computed document similarities to improve of ranking results in MlaS.
 - Evaluation, Evaluation, Evaluation.

Future Works – Gensim Math Document Similarities

- Gensim by Radim Řehůřek is “the most robust, efficient and hassle-free piece of software to realize unsupervised semantic modelling from plain text”: <http://radimrehurek.com/gensim/>
- We are experimenting with correlation of document similarities based on Mathematics Subject Classification (MSC) vs. document similarities based on formulae and other prominent parts (title, authors, abstract...) of the documents.
- Visualized similarity matrices.

Future Works – Gensim Math Document Similarities



Method: TfIdf-LSI; Weighted MTerms: true; MTerm Weight Conversion: 1

Future Works – Gensim Math Document Similarities

- MSC sorted documents in columns/rows, white lines separate MSC codes with different two characters (top category).
- All the documents compared each other.
 - Grayscale level indicates similarity of the document on the row to the document in the column.

white Very similar.

black Very different.

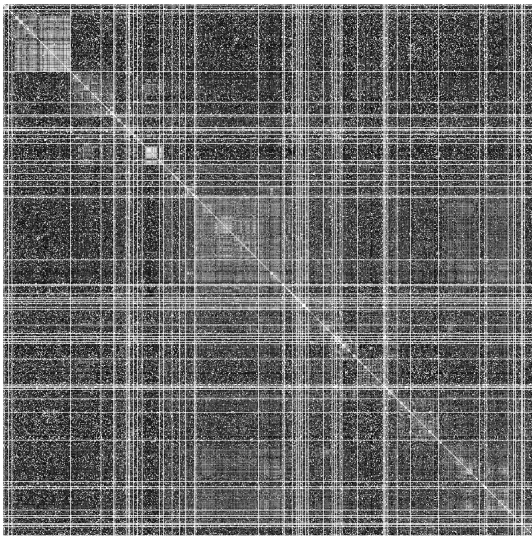
- Different weighting (or even omitting) of tokens from various metadata fields.
- Vector space model transformation method:

TfidfF-LSI Inverse Document Frequency wrapped by Latent Semantic Indexing

LDA Latent Dirichlet Allocation

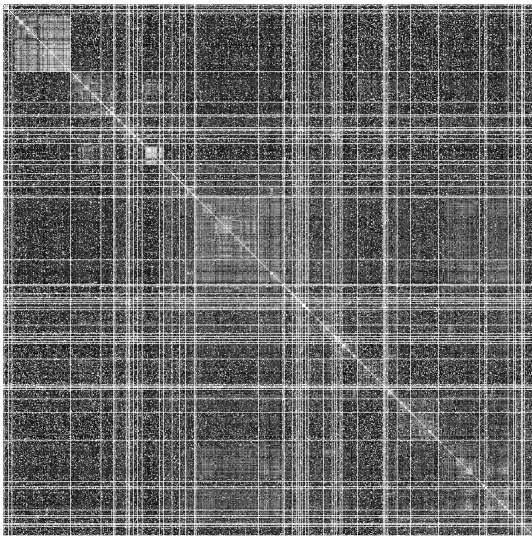
... ..

Future Works – Gensim Math Document Similarities



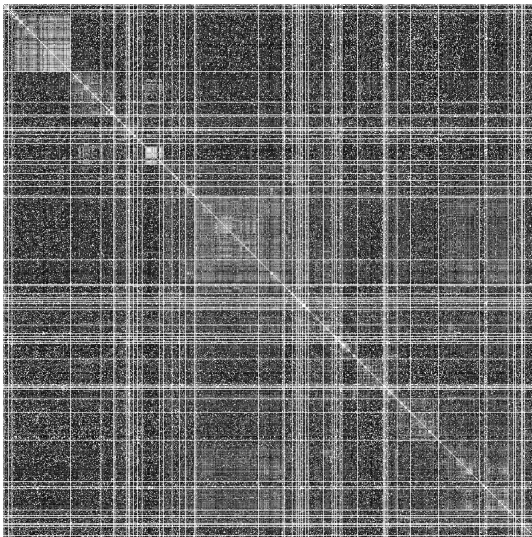
Method: TfIdf-LSI; Weighted MTerms: true; MTerm Weight Conversion: 1

Future Works – Gensim Math Document Similarities



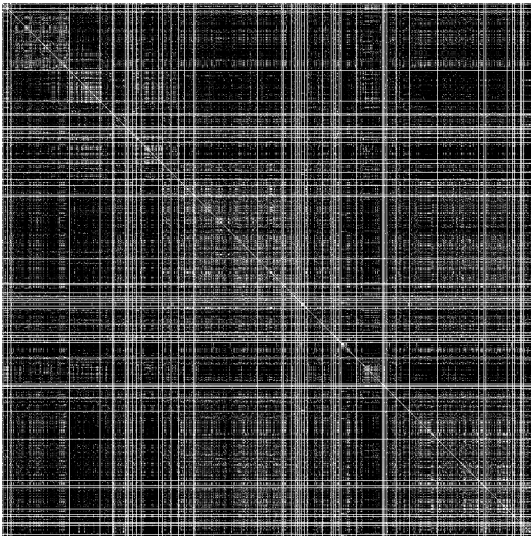
Method: TfIdf-LSI; Weighted MTerms: true; MTerm Weight Conversion: trunc(3.9 * mtermWeight)

Future Works – Gensim Math Document Similarities



Method: TfIdf-LSI; Weighted MTerms: false

Future Works – Gensim Math Document Similarities



Method: LDA; Weighted MTerms: false

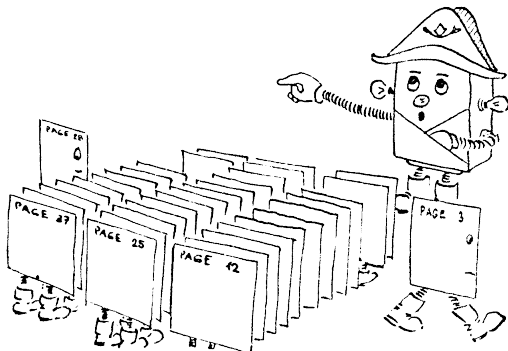
Future Works – Gensim Math Document Similarities

- Gensim by Radim Řehůřek is “the most robust, efficient and hassle-free piece of software to realize unsupervised semantic modelling from plain text”: <http://radimrehurek.com/gensim/>
- We are experimenting with correlation of document similarities based on Mathematics Subject Classification (MSC) vs. document similarities based on formulae and other prominent parts (title, authors, abstract...) of the documents.
- Visualized similarity matrices.
 - How to compute similarity of these matrices rigorously?
 - Canonical-correlation analysis (CCA)?

Summary

- We have our own math-aware search engine.
- We have a lot of possible improvements in our minds.
- We are interested in evaluation a lot.

Questions?





Illustrations by Jiří Franek.



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